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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 10
Question Paper Code : UM9279

KEY

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B	C	A	B	C	B	C	C	A	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	B	A	A	C	D	A	C	D	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	A	C	C	B	D	C	C	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
B, C	A,B,C,D	A,B,C,D	B,D	A,B,C,D	A	D	C	B	D
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C	C	D	D	C	C	C	A	D	C

EXPLANATIONS

MATHEMATICS - 1

1: (B) 0 + 0 = 0 × 0 and 2 + 2 = 2 × 2 (OR)

Given $x + y = xy \Rightarrow x = xy - y$

$x = y(x - 1)$

$$y = \frac{x}{x-1}$$

If $x = 0$ then $y = 0$ &

If $x = 2$ then $y = 2$

2: (C) In a cube diagonal length is longest

$\therefore \sqrt{3}\,l$ is the maximum distance between any two points

3: (A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} =$

$$\frac{a^2\sec^2\theta\cos^2\phi}{a^2} + \frac{b^2\sec^2\theta\sin^2\phi}{b^2} - \frac{c^2\tan^2\theta}{c^2}$$

$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$

$= \sec^2\theta - \tan 2\theta = 1$

4: (B) Time for one tick for first pendulum = $\frac{58}{57}$ seconds

Time for one tick for second pendulum = $\frac{609}{608}$ seconds

$\therefore$ Time taken to tick together = LCM of both

$= \frac{1218}{19}$ Seconds

5: (C) Given $\angle ACP = 60°$

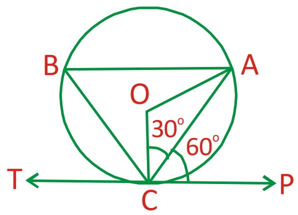
But $\angle OCP = 90°$

$\therefore \angle ACO = 90° - 60° = 30°$

In $\Delta AOC, OC = OA \Rightarrow \angle OCA = \angle OAC = 30°$

$\therefore \angle AOC = 180° - 30° - 30° = 120°$

$\therefore \angle ABC = \frac{1}{2} \angle AOC = \frac{120°}{2} = 60°.$



6: (B) $3x^2 + 17x + 24 = 3x^2 + 9x + 8x + 24$

$= 3x(x + 3) + 8(x + 3)$

$= (x + 3)(3x + 8)$

$(x + 3)\ \overline{)\ 6x^3 + 17x^2 - 5x - 6\ }\ 6x^2 - x - 2$

$6x^3 + 18x^2$
(–) (–)

$-x^2 - 5x - 6$

$-x^2 - 3x$
(+) (+)

$-2x - 6$

$-2x - 6$

0

$\therefore (x + 3)$ is the HCF of both polynomials

$\therefore x - k = x + 3$

k = –3

7: (C) Area of shaded region = Area of parallelogram ABCD – Area of quarter circle AOC – Area of ΔODC

$= (AO + OD) \times OC - \frac{1}{4} \times \pi \times (OA)^2 - \frac{1}{2} \times OD \times OC$

= (14 + 7) cm × 14 cm

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 cm^2 - \frac{1}{2} \times 7 \times 14 cm^2$

$= 21 \times 14 cm^2 - 154 cm^2 - 49 cm^2$

$= (294 - 154 - 49) cm^2$

$= 91 cm^2$

8: (C) Given a, b, c are in AP $\Rightarrow$ b = a+d & c = a + 2d

given $ax + by + c = 0$

$ax + (a + d)y + (a + 2d) = 0$

$ax + ay + dy + a + 2d = 0$

$a(x + y + 1) + d(y + 2) = 0$

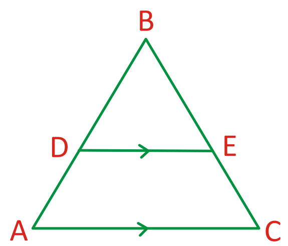
If $y = -2$ and $x = 1$ then given expression is zero

$\therefore ax + by + c = 0$ line passes through (1, –2)

9: (A) Corresponding sides ratio = Square root as areas ratio

$= \sqrt{\frac{15}{19}} = \sqrt{15} : \sqrt{19}$

10: (D)



Given DE // AC $\Rightarrow \Delta BDE \sim \Delta BAC$

$\therefore \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta BAC} = \frac{(BD)^2}{(AB)^2}$

$\Rightarrow \frac{\frac{1}{2} \text{area of } \Delta BAC}{\text{Area of } \Delta BAC} = \left(\frac{BD}{AB}\right)^2$

$\therefore \frac{BD}{AB} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \rightarrow 1$

$$\therefore\ 1-\frac{BD}{AB}=1-\frac{1}{\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AB-BD}{AB}=\frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AD+\cancel{DB}-\cancel{BD}}{AB}=\frac{\sqrt{2}-1}{\sqrt{2}} \quad \rightarrow 2$$

$$\frac{eq2}{eq1}\Rightarrow\frac{\left(\frac{AD}{\cancel{AB}}\right)}{\left(\frac{BD}{\cancel{AB}}\right)}=\frac{\left(\frac{\sqrt{2}-1}{\cancel{\sqrt{2}}}\right)}{\left(\frac{1}{\cancel{\sqrt{2}}}\right)}=\sqrt{2}-1$$

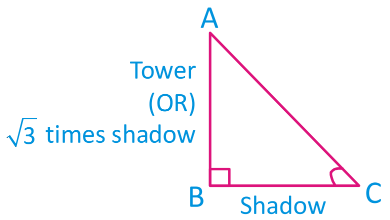
$$\therefore \frac{BD}{AD}=\frac{1}{\sqrt{2}-1}$$

11: (C) Given $\angle B = 90°$ & Let $\angle C = \theta$

$$\tan\theta=\frac{AB}{BC}=\sqrt{3}\frac{\cancel{shadow}}{\cancel{shadow}}$$

$\tan\theta = \sqrt{3}$ = tan 60°

$\therefore\ \theta = 60°$



12: (B) Volume of cube = $(7cm)^3$ = $343cm^3$

Diameter of cone = side of cube = 7cm

$\therefore\ r = \frac{7cm}{2}$ & h = 7cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$=\frac{1}{3}\times\frac{\cancel{22}^{11}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{2}}\times\frac{7}{2}\times 7cm^3$$

= 89.83 cm^3

wastage of wood = 343 cm^3 – 89.83cm^3 = 253.17cm^3

waste wood percentage

$$=\frac{253.17cm^3}{343cm^3}\times 100$$

= 73.81%

13: (A) Given $\alpha+\beta=\frac{-b}{a}$ & $\alpha\beta=\frac{c}{a}$

$$\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$=\sqrt{\frac{b^2}{a^2}-\frac{4c}{a}}$$

$$=\frac{\sqrt{b^2-4ac}}{a}$$

$$\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)=\frac{-b\sqrt{b^2-4ac}}{a^2}$$

14: (A) Given $3x^2 + 2x^2 + x - k = 0$

$\therefore$ $5x^2 + x - k + 5 = 0$

Given $\alpha + \beta = \alpha\beta$

$$\Rightarrow -\frac{b}{\cancel{a}}=\frac{c}{\cancel{a}}$$

–1 = – k + 5

–k = –6

k = 6

15: (C) In ΔABC, $\angle ACB = 90°$

[$\because$ AC is tangent to the circle of centre ‘B’]

$\therefore\ \angle A = 90° - 30° = 60°$

In ΔAPC, $\sin 60° = \frac{CP}{AC}$

$$\frac{\sqrt{3}}{2}=\frac{CP}{4cm}\Rightarrow\quad CP=2\sqrt{3}cm$$

$\therefore$ CD = 2CP = 2 × $2\sqrt{3}$cm = $4\sqrt{3}$cm

In ΔAPC, $\cos 60° = \frac{AP}{AC}$

$$\Rightarrow \frac{1}{2}=\frac{AP}{4cm}\Rightarrow AP=2cm$$

Area of shaded region = area of the sector ACD – area of ΔACD

$$=\frac{\cancel{120°}^{1}}{\cancel{360°}_{3}}\times\frac{22}{7}\times4\times4cm^2-\frac{1}{\cancel{2}}\times4\sqrt{3}\times\cancel{2}cm$$

= 16.76 cm^2 – 4 × 1.73 cm^2

= 16.76 cm^2 – 6.92 cm^2

= 9.84 cm^2

16: (D) Given $\frac{10}{2}[2a + 9d] = 4 \times \frac{5}{2}(2a + 4d)$

$2a + 9d = 10 \times \frac{2}{10}(2a + 4d)$

$2a + 9d = 4a + 8d$

$2a - 4a = 8d - 9d$

$-2a = -d$

$\frac{a}{d} = \frac{1}{2}$

$\therefore$ a : d = 1 : 2

17: (A) HCF of 108 & 204 = 12.

18: (C) Area of plate = $\pi R^2 = \pi \times 20 \times 20\ cm^2 = 400\ \pi\ cm^2$

Area of cut portion = $4 \times \pi r^2 = 4 \times \pi \times 5 \times 5\ cm^2 = 100\ \pi\ cm^2$

Area of uncut portion = $400\pi cm^2 - 100\ \pi cm^2 = 300\ \pi cm^2$

Ratio of uncut portion and cut portion

$= \cancel{300\pi}^{3} : \cancel{100\pi}^{1} = 3 : 1$

19: (D) Let Raju's present age be x years and Ayan's present age be 'y' years

Given $x + \frac{y}{2} = 14$

$\frac{2x + y}{2} = 14$

$2x + y = 28 \to 1$

$\frac{x}{3} + 2y = 34$

$\frac{x + 6y}{3} = 34$

$x + 6y = 102 \to 2$

eq2 × 2 $\Rightarrow$ $\cancel{2x} + 12y = 204$
$\cancel{2x} + y = 28$
(–) (–) (–)

$\cancel{11}y = \cancel{176}^{16}$

$y = 16$

$2x + 16 = 28 \to 1$

$2x = 12$

$x = 6$

$x + y = 16 + 6 = 22$

20: (A) $x^2 + x + 1 = 0$ has no real roots is true option

21: (C) Given $a_3 = a + 2d = 600 \to 1$

$a_7 = a + 6d = 720 \to 2$

eq 2 – 1 $\Rightarrow$ 4d = 120

$d = \frac{120}{4} = 30$

$\therefore$ a + 2(30) = 600

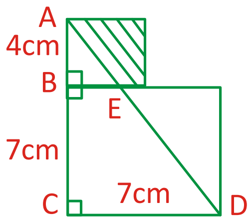
a = 540

$S_7 = \frac{7}{2}[2a + 6d] = \frac{7}{\cancel{2}} \times \cancel{2}[a + 3d]$

= 7 [540 + 90]

= 7 × 630 = 4410

22: (A)



$\Delta ABE \sim \Delta ACD$

$\therefore \frac{AB}{AC} = \frac{BE}{CD}$

$\Rightarrow \frac{4cm}{11cm} = \frac{BE}{7cm}$

$\therefore$ BE = $\frac{7 \times 4cm}{11} = \frac{28cm}{11}$

Area of ΔABE

$= \frac{1}{2} \times AB \times BE = \frac{1}{\cancel{2}_1} \times \cancel{4}^2 cm \times \frac{28}{11} cm$

$= \frac{56cm^2}{11}$

Area of shaded region = $(4cm)^2 - \frac{56}{11} cm^2$

$= \frac{176cm^2 - 56cm^2}{11}$

$= \frac{120}{11} cm^2$

23: (A) HCF of

$$\frac{14}{3},\frac{21}{5} \& \frac{7}{15} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{7}{15}$$

24: (C) Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{\cancel{7}_1} \times \cancel{14}^{2} \times 14 \text{cm}^2$$

$= 2464 \text{ cm}^2$

$\therefore$ Total cost for painting

$$= 2464 \cancel{\text{cm}^2} \times \frac{20\text{paise}}{1 \cancel{\text{cm}^2}}$$

= 49280 paise

= ₹ 492.8

25: (C) Given $\alpha + \beta = 19$ & $\alpha - \beta = 5 \Rightarrow \alpha = 12$ & $\beta = 7$

$\therefore$ Required quadratic polynomial

$= x^2 - x(\alpha + \beta) + \alpha\beta = 0$

$\Rightarrow x^2 - 19x + 84 = 0$

26: (B) Let $\frac{1}{\sqrt{x}} = a \,\&\, \frac{1}{\sqrt{y}} = b$

$\Rightarrow 2a + 3b = \frac{13}{6}$ and $4a - 9b = -\frac{19}{6}$

$\Rightarrow 6(2a + 3b) = 13$ $6(4a - 9b) = -19$

$12a + 18b = 13 \rightarrow 1$ $24a - 54b = -19 \rightarrow 2$

eq $1 \times 2 \Rightarrow$

$$\begin{array}{rcrl} \cancel{24a} - 54b &=& -19 & \longrightarrow 2 \\ \cancel{24a} + 36b &=& 26 & \\ (-)\quad (-) & & (-) & \\ \hline \cancel{+}90b &=& \cancel{+}45 & \end{array}$$

$$b = \frac{45}{90} = \frac{1}{2}$$

$$12a + \cancel{18}^{9}\left(\frac{1}{\cancel{2}}\right) = 13 \rightarrow 1$$

$12a = 13 - 9 = 4$

$$a = \frac{\cancel{4}^{1}}{\cancel{12}_3} = \frac{1}{\sqrt{x}} \;\&\; b = \frac{1}{2} = \frac{1}{\sqrt{y}}$$

$\therefore \sqrt{x} = 3 \,\&\, \sqrt{y} = 2$

$\therefore x = 9 \,\&\, y = 4$

$x + y = 13$

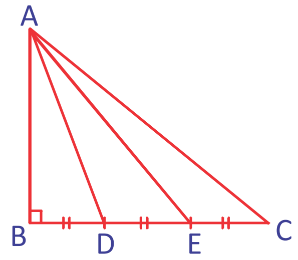
27: (D) Given $a_n = 2n + 1$

$\therefore a_8 = 2(8) + 1 = 17$

$a_{15} = 2(15) + 1 = 31$

$\therefore a_8 + a_{15} = 17 + 31 = 48$

28: (C) In ΔABC, $\angle B = 90°$ & BD = DE = EC



$\therefore$ $3AC^2 + 5AD^2 = 3(AB^2 + BC^2) + 5(AB^2 + BD^2)$

$= 3AB^2 + 3BC^2 + 5AB^2 + 5BD^2$

$$= 8AB^2 + 3\left(\frac{3BE}{2}\right)^2 + 5\left(\frac{BE}{2}\right)^2$$

$$[\therefore BC = BE + EC = BE + \frac{BE}{2} = \frac{3BE}{2}]$$

$$= 8AB^2 + \frac{27BE^2}{4} + \frac{5BE^2}{4}$$

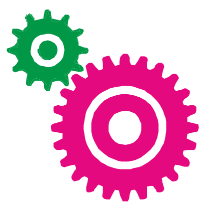
$$= 8AB^2 + \frac{27BE^2 + 5BE^2}{4}$$

$$= 8AB^2 + \frac{\cancel{32}^{8}BE^2}{\cancel{4}_1}$$

$= 8(AB^2 + BE^2)$

$= 8AE^2$

29: (C) Distance covered for one revolution of big wheel = $\pi D = 50\,\pi$cm



Distance covered for 15 revolution of big wheel = $15 \times 50\,\pi$ cm = $750\,\pi$ cm

Distance covered by small wheel in one revolution = $\pi d = 30\pi$ cm

$\therefore$ No. of revolutions required for small wheel to cover 750πcm} = $\frac{750\pi}{30\pi}$ = 25

= 25

30: (D) Given radius of cone = $\frac{14cm}{2}$ = 7cm & h = 8cm

$\therefore$ Volume of cone = $\frac{1}{3}\pi \times 7 \times 7 \times 8cm^3$

$\therefore \frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi \times 7 \times 7 \times 8^{2} cm^3$

$\Rightarrow 5^3 - r^3 = 98cm^3$

$(125 - 98)cm^3 = r^3$

$r^3 = 27cm^3 = (3cm)^3$

$\therefore$ d = 2r = 6 cm

MATHEMATICS - 2

31: (B, C)

Given $a_1b_2 = a_2b_1 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ coinsiding lines

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ parallel lines

32: (A,B,C,D)

$a + b = 3 - \sqrt{2} + 3 + \sqrt{2} = 6$ which is a rational number

$a - b = (3 - \sqrt{2}) - (3 + \sqrt{2}) = 3 - \sqrt{2} - 3 - \sqrt{2} =$

$-2\sqrt{2}$ which is an irrational number.

$ab = (3 - \sqrt{2})(3 + \sqrt{2}) = 3^2 - (\sqrt{2})^2 = 9 - 2 = 7$ which is a rational number

$$\frac{a}{b} = \frac{3-\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{(3-\sqrt{2})^2}{9-2} = \frac{9-6\sqrt{2}+2}{7} = \frac{11-6\sqrt{2}}{7}$$

which is an irrational number. Hence it is real number

33: (A, B, C, D)

$m^3 - m = m(m^2 - 1^2)$

$= m(m + 1)(m - 1)$

$= (m - 1)(m)(m + 1)$

Product of three consecutive numbers are always divisible by 6

$\therefore$ $(m^3 - m)$ is also divisible by the factors of 6

ie 1, 2, 3 & 6

34: (B, D)

$x^2 - 10x + 18 = 0$

a = 1 b = – 10 c = 18

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-10) + \sqrt{100 - 4 \times 18 \times 1}}{2}$$

$$= \frac{10 \pm \sqrt{28}}{2} = \frac{10 \pm 2\sqrt{7}}{2}$$

$= 5 \pm \sqrt{7}$

35. (A, B, C, D)

Sum of the first 120 natural numbers

$$= \frac{n(n+1)}{2} = \frac{120 \times 121}{2}$$

= 60 × 121

30 is a factor of 60 × 121

11 is a factor of 60 × 121

165 is a factor of 60 × 121

44 is a factor of 60 × 121

REASONING

36. (A) A —+2→ C —+2→ E —+2→ G

O —+2→ Q —+2→ S —+2→ U

37. (D) Step 1: Dark shaded triangle is move 2 steps anticlock direction

Step 2: the square in the shape is moved 2 steps anticlock direction

Step 3: the circle is move one step clockwise direction and it is changed to dark and light shades every alternate shape

38. (C) C@B implies C is the sister of B. B%F implies B is the son of F. Hence C is the daughter of F. F%E implies F is the son of E. Hence, C is the granddaughter of E. Hence option C is the answer

39. (B) Each segment in the lower right box equals the sum of the values in the corresponding segments of the other squares.

40. (D) Second image is the top view of first image. Similarly fourth image is the top view of third image.

41. (C) All the figures have a house shape with a chevron inside. All the shading is the same. Each figure also has an 'L' shape outside the main shape, and because two are on the left and two on the right, this cannot be what makes an odd one out. Counting the number of straight lines pointing down, you will see that C has 9, but A, B and D all have 8

42. (C) 29

43. (D) Milan takes a left turn while facing East, so now he faces in the North direction

Option A: three left turns from north would lead him to facing east

Option B: three right turns from north would lead him to facing west

Option C: one left turn from north would lead him to face West

Therefore both options B and C are correct

44. (D) Friday

Three days after Saturday is Tuesday.

The day before the day before yesterday is Tuesday.

Since the day before, the day before yesterday is Tuesday.

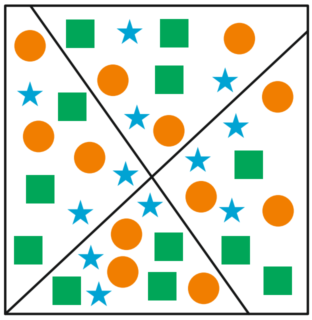
Today must be Friday.

45. (C) dc = 32; f = 5; bf = 15; d = 3

124 = bce

CRITICAL THINKING

46. (C)

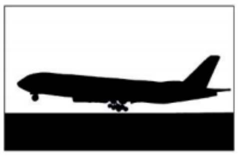


47. (C) Reason is incorrect. Haemoglobin has high affinity for oxygen.

48. (A) (i) and (iv) In the given shape the outer shape are corner is exactly between the inner shape site.

49. (D) If the data given in both statements I and II together are not sufficient to answer the question.

50. (C)



Plane is landing smothly comparing with option (A), (B), (D).